Fifth Wheel Modelling and Testing

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Abstract

A two dimensional dynamic model, which provides a full description of a vehicle motion based on sensory input, is presented. The model has three sensory inputs: the velocity of a fifth wheel; the orientation of the fifth wheel with respect to the vehicle; and the yaw rate of the vehicle. Based on these inputs, the model determines the position, velocity, acceleration and the yaw angle of the vehicle. The results of experiment with an instrumented vehicle, which were designed to verify the validity and accuracy of the model, are reported.

NOMENCLATURE AND NOTATIONS

Coordinate systems

R_O (O, $\hat{x}_0$, $\hat{y}_0$, $\hat{z}_0$) - a global frame of reference

R_C (G, $\hat{x}_C$, $\hat{y}_C$, $\hat{z}_C$) - a frame attached to the vehicle's center of gravity

R_P (P, $\hat{x}_P$, $\hat{y}_P$, $\hat{z}_P$) - a frame attached to the 5th wheel at the pivot point

Angles

Ψ - angle between R_O and R_C measured about $\hat{z}_0$

Θ - angle between R_C and R_P measured about $\hat{z}_0$

β_P - angle between $\vec{V}_{P\in C/O}$ and x_C measured about $\hat{z}_0$

Points

G - vehicle's center of gravity
W - center of the 5th wheel axis of rotation
P - 5th wheel pivot point
R_1 - center of the left rear wheel
F_1 - center of the left front wheel

Subscripts

C - the vehicle
P - the 5th Wheel assembly

Vector notation example

$$\vec{V}_{G\in C/O} = \left|\begin{matrix} u_G \\ v_G \\ 0 \end{matrix}\right._{R_O}$$

- Velocity of the vehicle' center of gravity, G, which is located on the vehicle, C, with respect to frame R_O.

I. INTRODUCTION

Automotive 5th wheels have been used for decades in vehicle testing. The 5th wheel is attached to the rear bumper is towed by the vehicle. The attachment lows relative rotation between the vehicle's and the wheel's axes. An optical encoder attached to the wheel's axis reads the angular velocity of the wheel. Knowing the diameter of the wheel, assuming no slip, the vehicle vehicle's speed can be determined. Usually the 5th wheel is being used to test the performance of the braking systems, to measure coefficient of friction between the tires and road surface, vehicle's acceleration etc. Obviously, in these applications the vehicle has to travel along a straight line.

Other sensors such as accelerometers and gyroscopes are also being installed on vehicles in order to extract information about their dynamic behaviour.

This project deals with high yaw rate manoeuvres such as sharp turning, high speed passing and side impacts at high speed. Therefore there is a need to determine the yaw rate of the vehicle and its speed. Therefore, a conventional 5th wheel, which is instrumented only with encoder, is not sufficient and additional sensors are required.

A commercial 5th wheel was modified to include a sensor at its pivot point which measures the angle between the 5th wheel and the vehicle axes. Also, a gyroscope, measuring the vehicle's yaw rate, was installed. The location of the sensors and the 5th wheel are shown in Figure 1.

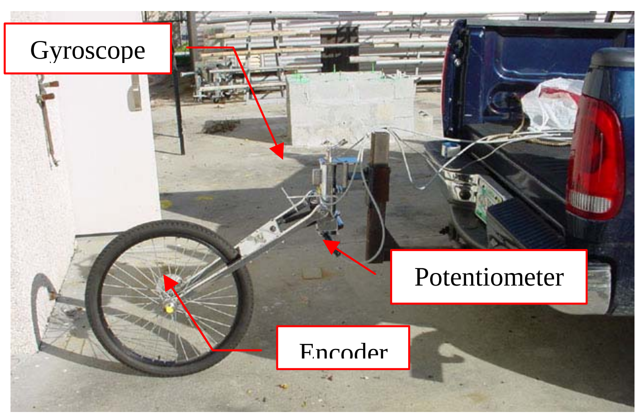


Figure 1: The fifth wheel and the sensors.

II. SENSORS AND SIGNAL PROCESSING

The gyroscope

A gyroscope, that measures the vehicle's yaw rate, $\dot{\Psi}$, was attached to the vehicle close to the pivot point, P. A very low constant speed electrical motor was used to calibrate the gyroscope. The gyroscope was attached to the motor shaft and the motor was excited with different voltages to produce different measurable angular velocities. Using the known velocities and the corresponding gyroscope outputs a calibration curve was established:

$$\dot{\Psi}[\frac{\circ}{s}] = 49.92\,\dot{\Psi}[\mathrm{V}] - 115.08 \qquad (1)$$

The encoder

The encoder, which measures the fifth wheel speed, u_W, was attached to the wheel's axis. Its digital signal was converted to an analogue signal proportional to the wheel speed, u_W. The wheel speed was calibrated using the electronic display that was provided with the wheel. The conversion of the analogue signal to wheel velocity found to be:

$$U_W\,[\frac{\mathrm{m}}{\mathrm{s}}] = 8.7729\,U_W\,[\mathrm{V}] - 0.0428 \qquad (2)$$

The Potentiometer

The potentiometer measures the angle, Θ, between the 5th wheel and the vehicle axes. It was calibrated manually using a protractor:

$$\Theta[^\circ] = -71.667\,\Theta[\mathrm{V}] + 177.4 \qquad (3)$$

Filtering the signals

First, out liar readings are detected by comparing the current sampled value to the average of the last *m* readings. If the difference exceeds a given threshold the reading is removed and replaced in order to keep the time track valid:

$$if \quad \left(\left[\left| \frac{1}{m} \sum_{j=i-m}^{i-1} X_j - X_i \right| \geq C \right] \right) \quad then$$
$$X_i = \frac{1}{n} \sum_{k=i-n}^{i-1} X_k \qquad (4)$$

where X_i is the measured value and *m* and *n* are integers.

Second a local linearization was performed by fitting the sampled reading to a straight line defined by the previous 10 samples and the next 10 samples (see Figure 2).

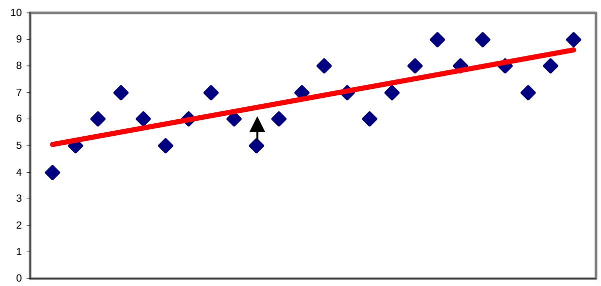
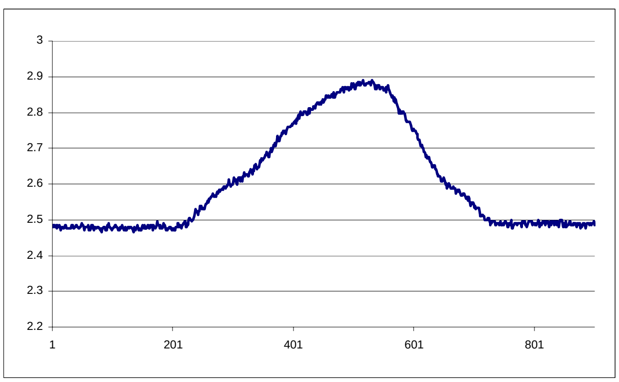
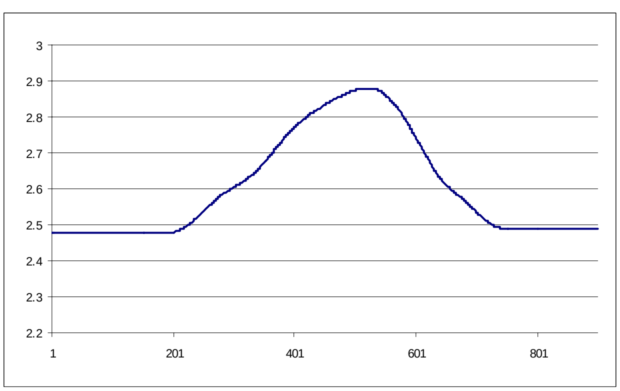


Figure 2: Local linearization.

The first operation was repeated with threshold value equal to the standard deviation of the next *q* readings. An example for the above process is shown in Figure 3.



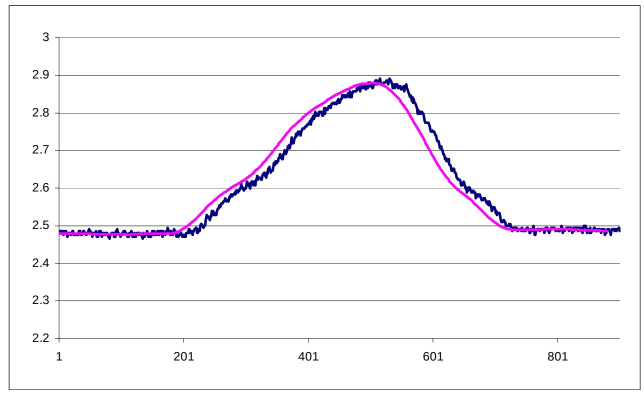
Original signal



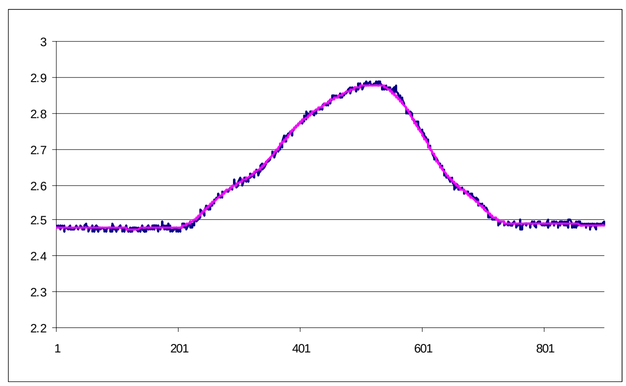
Filtered signal

Figure 3: Results of the filtering operations.

The above processing introduced delay between the original and the processed signal, as shown in Figure 4. Since the signal has to be integrated with respect to time, the filtered signal was shifted back in time to match the original one. Figure 4 illustrates this operation.



Original and filtered signal



Filtered signal shifted back in time

Figure 4: Shifting the filtered signal.

Integration and derivatives

As will be shown in the proceeding section the integration and derivative of the signal might be needed. Simple algorithms were used for both operations:

$$f'(n) = f_s * (X_{n+1} - X_n) \tag{5}$$

$$\int_0^t x(t)dt \cong \frac{1}{f_s}\sum_{i=0}^{n} X_i(i) \tag{6}$$

where f_s is the sampling rate.

III. MODELLING

The purpose of the following section is to develop a 2D model which will describe the vehicle's motion based on the available real time sensory input. In this case the velocity of the vehicle, defined by the components u_G and v_G should be determined by the measured values of the 5th wheel speed, u_W, its pivot angle, Θ, and the vehicle angular velocity, $\dot{\Psi}$.

The dimensions coordinate systems and velocities of the vehicle and the 5th wheel are shown in Figures 5 and 6.

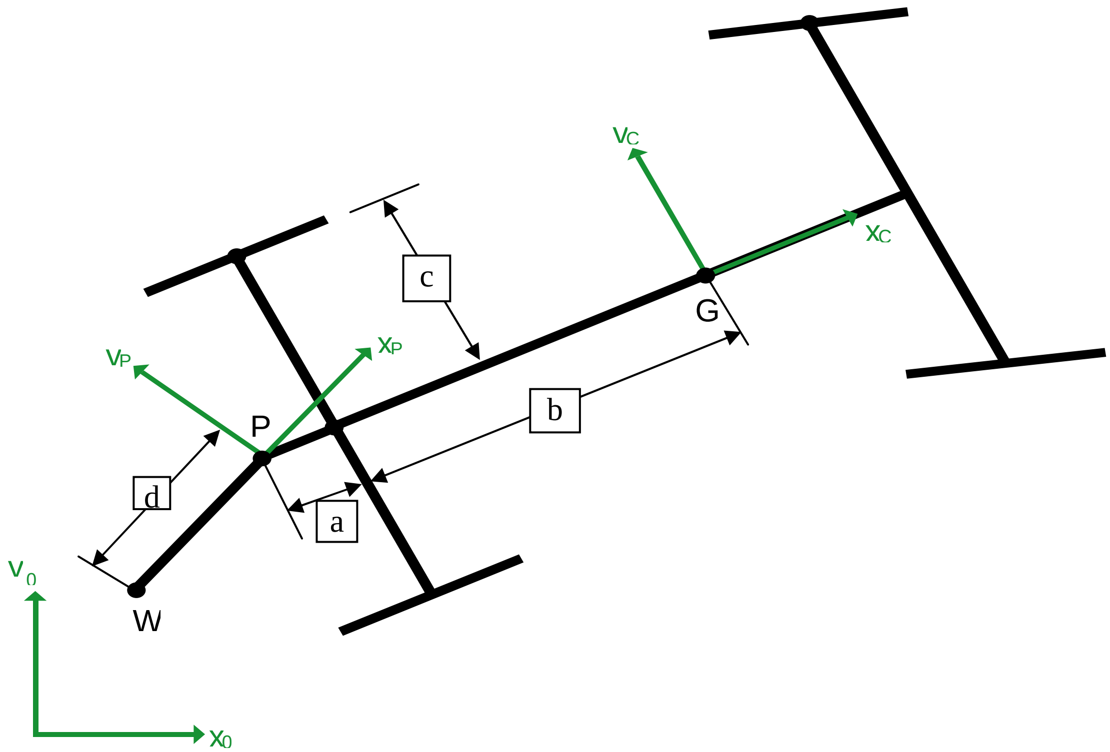


Figure 5: Dimensions of the vehicle and the 5th wheel.

The position of Fifth wheel axis, W, with respect to R_O is given by:

$$\overrightarrow{OW} = \overrightarrow{OP} - \overrightarrow{WP} \tag{7}$$

where these vectors are given by:

$$\overrightarrow{OW} = \left| \begin{matrix} x_W \\ y_W \\ 0 \end{matrix} \right._{R_O} \quad \overrightarrow{OP} = \left| \begin{matrix} x_P \\ y_P \\ 0 \end{matrix} \right._{R_O} \quad \overrightarrow{WP} = \left| \begin{matrix} d\cos(\Theta+\Psi) \\ d\sin(\Theta+\Psi) \\ 0 \end{matrix} \right._{R_O} \tag{8}$$

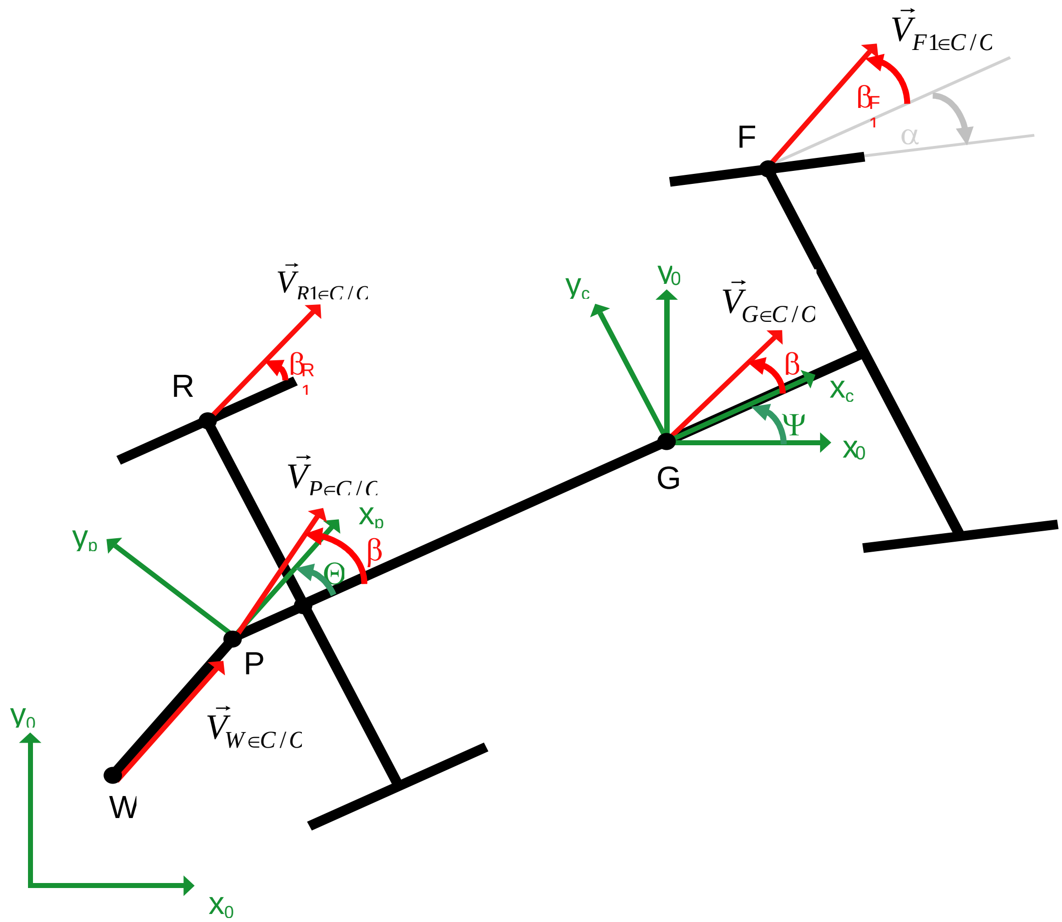


Figure 6: Velocities and angles definitions.

Thus, the position of the fifth wheel axis is given by:

$$\begin{cases} x_W = x_P - d\cos(\Theta+\Psi) \\ y_W = y_P - d\sin(\Theta+\Psi) \end{cases} \tag{9}$$

The velocity of the fifth wheel with respect to R_O is given by:

$$\vec{V}_{W\in P/O} = \left| \begin{matrix} u_W \\ 0 \\ 0 \end{matrix} \right._{R_P} = \left| \begin{matrix} u_W\cos(\Theta+\Psi) \\ u_W\sin(\Theta+\Psi) \\ 0 \end{matrix} \right._{R_O} \tag{10}$$

deriving the velocities from Eq. 3:

$$\begin{cases} \dot{x}_W = \dot{x}_P + d(\dot{\Theta}+\dot{\Psi})\sin(\Theta+\Psi) \\ \dot{y}_W = \dot{y}_P - d(\dot{\Theta}+\dot{\Psi})\cos(\Theta+\Psi) \end{cases} \tag{11}$$

Substituting in Eq. 1 yields:

$$\begin{cases} u_W\cos(\Theta+\Psi) = \dot{x}_P + d(\dot{\Theta}+\dot{\Psi})\sin(\Theta+\Psi) \\ u_W\sin(\Theta+\Psi) = \dot{y}_P - d(\dot{\Theta}+\dot{\Psi})\cos(\Theta+\Psi) \end{cases} \tag{12}$$

Similarly, the position of the vehicle's center, G, of gravity, with respect to R_O is given by:

$$\overrightarrow{OG} = \overrightarrow{OP} + \overrightarrow{PG} \tag{13}$$

or

$$\begin{cases} x_G = x_P + (a+b)\cos(\Psi) \\ y_G = y_P + (a+b)\sin(\Psi) \end{cases} \quad (14)$$

and the velocity of the vehicle's center of gravity, G, is given by:

$$\begin{cases} \dot{x}_G = \dot{x}_P - \dot{\Psi}(a+b)\sin(\Psi) \\ \dot{y}_G = \dot{y}_P + \dot{\Psi}(a+b)\cos(\Psi) \end{cases} \quad (15)$$

or:

$$\begin{cases} u_G = -\dot{\Psi}(a+b)\sin(\Psi) + u_W \cos(\Theta+\Psi) \\ \quad - d(\dot{\Theta}+\dot{\Psi})\sin(\Theta+\Psi) \\ v_G = \dot{\Psi}(a+b)\cos(\Psi) + u_W \sin(\Theta+\Psi) \\ \quad + d(\dot{\Theta}+\dot{\Psi})\cos(\Theta+\Psi) \end{cases} \quad (16)$$

Eqs. 15 and 16 describe the 2D motion of the vehicle in term of the measured variables.

The yaw angle, β, of the vehicle is important since it provides indication whether or not the vehicle is skidding. In the following an expression for the yaw angle as function of the measured variables, u_W, Θ and $\dot{\Psi}$, will be determined. The velocity of the vehicle's center of gravity, G, with respect to R_O is given by:

$$\begin{aligned} \vec{V}_{G\in C/O} &= \vec{V}_{P\in C/O} + \overrightarrow{GP} \times \vec{\Omega}_{C/O} \\ &= \vec{V}_{W\in P/O} + \overrightarrow{PW} \times \vec{\Omega}_{P/O} \\ &\quad + \overrightarrow{GP} \times \vec{\Omega}_{C/O} \end{aligned} \quad (17)$$

Substituting the known distances yields:

$$\vec{V}_{G\in C/O} = \left| \begin{matrix} u_W \cos(\Theta) - d(\dot{\Theta}+\dot{\Psi})\sin(\Theta) \\ u_W \sin(\Theta) + d(\dot{\Theta}+\dot{\Psi})\cos(\Theta) \\ +(a+b)\dot{\Psi} \\ 0 \end{matrix} \right._{R_C} \quad (18)$$

And the yaw angle, β, is given by:

$$\beta = Arc\tan\left(\frac{u_W \sin(\Theta) + d(\dot{\Theta}+\dot{\Psi})\cos(\Theta) + (a+b)\dot{\Psi}}{u_W \cos(\Theta) - d(\dot{\Theta}+\dot{\Psi})\sin(\Theta)} \right) \quad (19)$$

VI. ERRORS ANALYSIS

Since u_W, Θ and $\dot{\Psi}$ are being measured, and the terms cosθ, sinθ, Ψ and $\dot{\theta}$ are used in the above relationships, it is essential to determine the affect of the measurements errors on the calculated value of the vehicle's center gravity velocity and position. The theoretical values of u_W, Θ and $\dot{\Psi}$ are given by:

$$\begin{aligned} u_W &= \tilde{u}_W + \varepsilon_W \\ \Theta &= \tilde{\Theta} + \varepsilon_\Theta \\ \dot{\Psi} &= \tilde{\dot{\Psi}} + \varepsilon_{\dot{\Psi}} \end{aligned} \quad (20)$$

where $\tilde{u}_W$, $\tilde{\Theta}$ and $\tilde{\dot{\Psi}}$ are the measured values and ε_W, ε_Θ and $\varepsilon_{\dot{\Psi}}$ are the corresponding measurement errors.

If Ψ contains bias error, it will accumulate due to time integration as:

$$\Psi = \int (\tilde{\dot{\Psi}} + \varepsilon_{\dot{\Psi}})dt \approx \tilde{\Psi} + t\varepsilon_{\dot{\Psi}} = \Psi + \varepsilon_\Psi \quad (21)$$

where ε_Ψ is the error in Ψ at the end of the integration period.

Differentiation of a signal that contains a bias error will not cause an error. However, differentiation of a noisy signal might produce large error and therefore it has to be filtered to some acceptable level.

The execution of trigonometry function may lead to errors:

$$\begin{aligned} \sin(\Psi) &= \sin(\tilde{\Psi} + \varepsilon_\Psi) \approx \sin(\Psi) + \cos(\Psi)\varepsilon_\Psi \\ \cos(\Psi) &= \cos(\tilde{\Psi} + \varepsilon_\Psi) \approx \cos(\Psi) - \sin(\Psi)\varepsilon_\Psi \end{aligned} \quad (22)$$

To reduce the errors, due to integration, in the position of the vehicle'' center of gravity, only the necessary integrations are performed:

$$
\begin{cases}
x_G = (a+b)\cos(\Psi) + \int [u_W \cos(\Theta+\Psi)]dt \\
\quad + d\sin(\Theta+\Psi) \\
v_G = (a+b)\sin(\Psi) + \int [u_W \sin(\Theta+\Psi)]dt \\
\quad + d\cos(\Theta+\Psi)
\end{cases}
\tag{23}
$$

V. EXPERIMENTS

Three different trajectories were used to validate the model and to test the sensory system. These trajectories are shown in Figure 7.

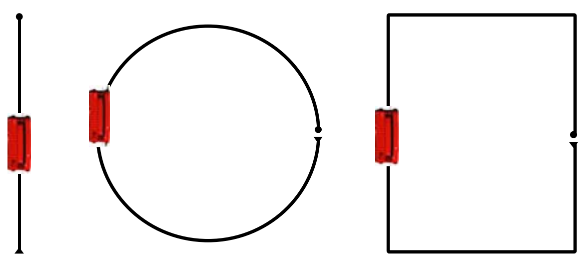


Figure 7: Trajectories used in the experiments.

Straight line trajectory

A 200 ft long straight line trajectory was used for this test. It took about 25 seconds to drive along this line, during which the sensors were sampled at 100 Hz (see Figure 8). The model used to determine the position and orientation of the vehicle. The longitudinal error was about 6 ft and the maximum lateral error was about 2 ft. These are very good results considering the fact the signals were integrated for a period of 25 seconds.

The noise at the beginning and at the end of the yaw angle (see Figure 9) is due to the division used in the model. At the beginning and the end of the trajectory both components of the velocity are very small and the presence of some noise will cause large errors.

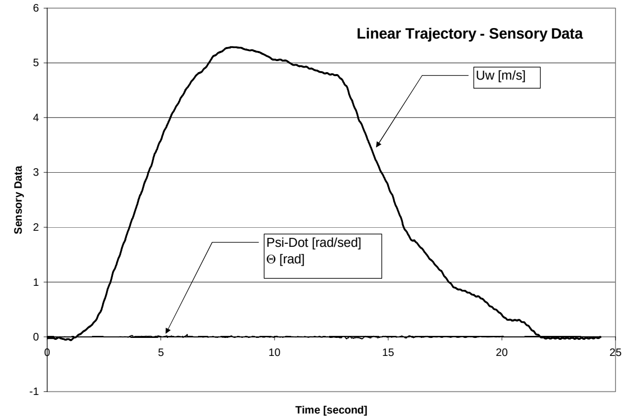


Figure 8: Straight line- sensory data.

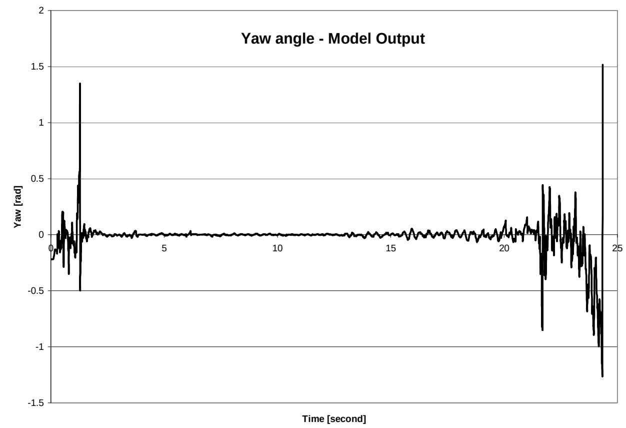


Figure 9: Yaw angle.

Square trajectory

The square trajectory was driven continuously four times. The sensory data obtained for one square are shown in Figure 10.

The trajectory, determined by the model, is shown in Figure 11. The errors in the trajectory are due to poor control of the driving track, rather than the model itself.

It is interesting to notice the Yaw angle of the vehicle along the trajectory. Even though driving speed was low, about 9 [mph], the vehicle was skidding at the corners of the square as indicated in figure 12 when ever the Yaw angle is nor zero.

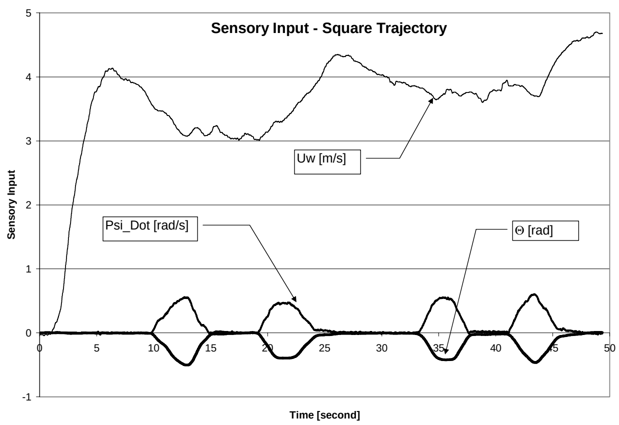


Figure 10: Square trajectory - sensory data.

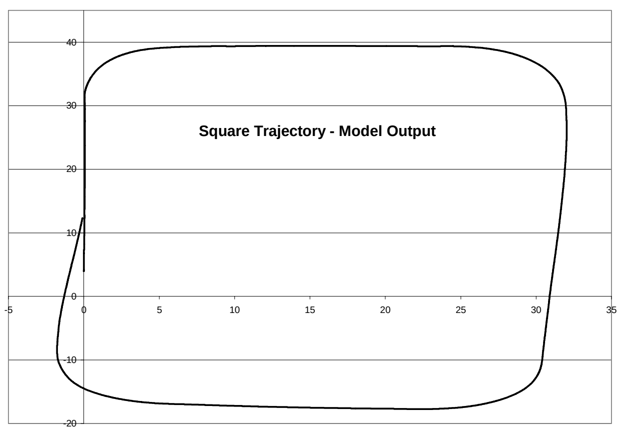


Figure 11: Square trajectory as constructed by the model.

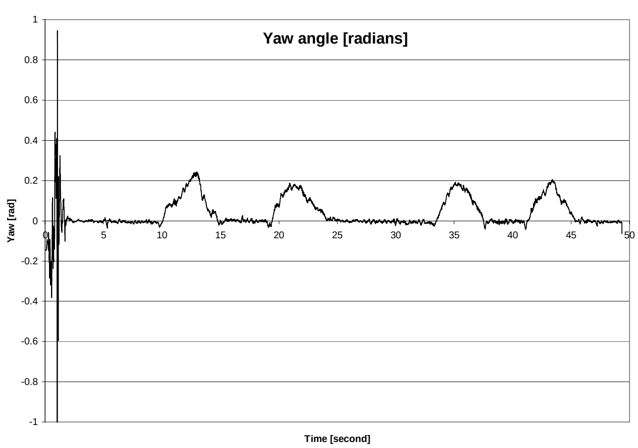


Figure 12: Yaw angle along the square trajectory.

Circular trajectory

The sensory inputs to the model, as recorded along the trajectory, are shown in Figure 12, and the constructed trajectory in Figure 13.

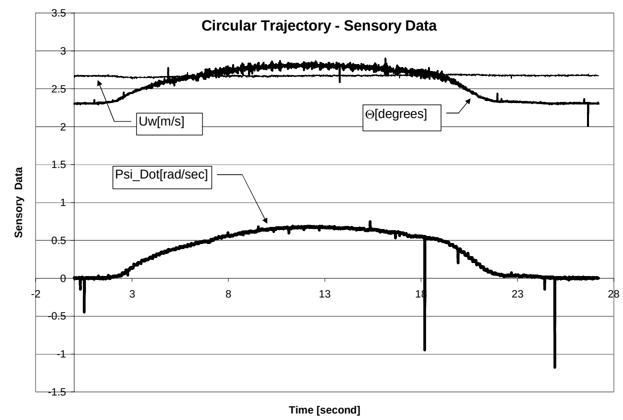


Figure 12: Circular trajectory – sensory data.

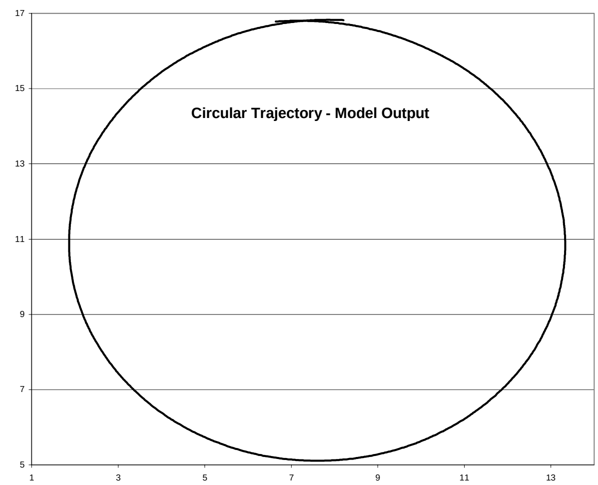


Figure 13: Circular trajectory constructed by the model.

In case of traveling in a constant speed along a circle the yaw angle should be constant. In this case, as shown in Figure 14, the yaw angle is not constant as expected, due to variations in the vehicle speed as shown in Figure 15.

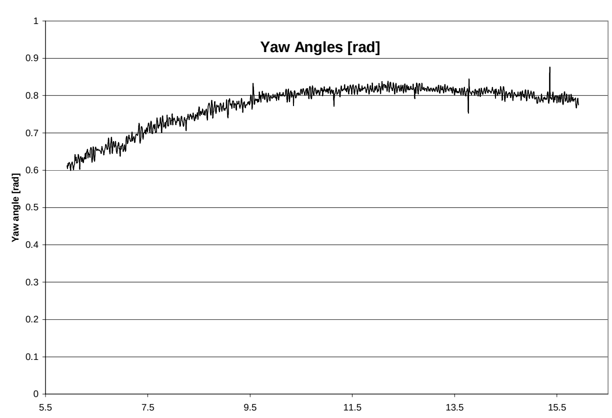


Figure 14: Yaw angle along the circle.

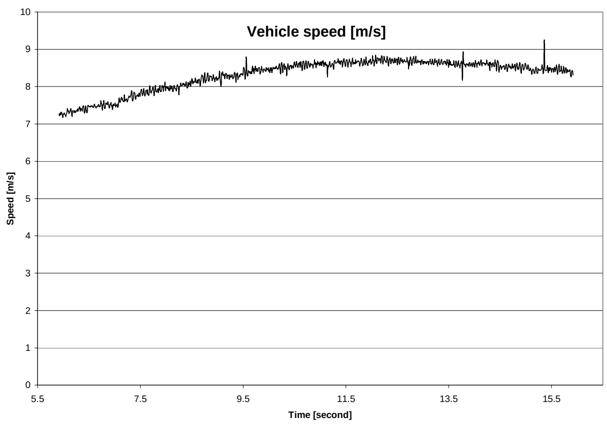


Figure 15: vehicle speed along the circle

VI. CONCLUSIONS

A simple, sensory based, model for 2D motion of a vehicle was introduced. Initial tests show good results. Better controlled experiments with additional sensors are planned in order to further evaluate the performance of the current sensors and the model results.