Cooperative Manipulation Testbed Development – Kinematics

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Abstract

This paper describes the development of a testbed for cooperative manipulation for dual-arm robots. Anthropomorphic maneuvering similar to human scale and dexterity using two manipulators to perform cooperative manipulation tasks are being developed in a Cooperative Manipulation Testbed comprised of two robotic manipulators. The testbed is described that can be used to investigate complex problems associated with dual-arm robotics.

The testbed includes two Staubli RX-60 CR clean room robots. The forward and inverse kinematics for the six degree-of-freedom manipulators is provided. The manipulators are being integrated to work cooperatively and perform similar functions to humans in both scale and dexterity. A corresponding simulation environment is also being developed with the testbed for off-line experimentation of algorithms. The testbed development strategy is also discussed including the software platform and architecture for the Cooperative Manipulation Testbed.

I. INTRODUCTION

Cooperative manipulation is an important enhancement to robotic capabilities as a dual-arm robot is enabled to perform more complex tasks, manipulate greater payloads, and span a greater workspace. The tasks performed by cooperating manipulators can be achieved in either teleoperated, semi-autonomous, and/or autonomous fashion.

The initial development of a Cooperative Manipulation Testbed is described that can be used to investigate the complex scenarios in addition to the other problems associated with implementation of dual-arm robotics. A corresponding simulation environment is being developed with the testbed for off-line experimentation of algorithms, particularly the collision avoidance algorithms. The requirements for the testbed are initially emphasized. The testbed development strategy is discussed including aspects of software and hardware. The requisite tooling, end-effectors, and mock-up environments are discussed. Software platform and architecture for the testbed, and corresponding simulation development environment for the tested are also described. Finally, an initial approach to the kinematics of the robots used in the Cooperative Manipulation Testbed is provided.

II. BACKGROUND

Several additional problems are posed by using a pair of robotic manipulators beyond the already complex problems associated with cooperative manipulation [1]. First of all, the manipulators work in close proximity to one another, therefore they must perform collision avoidance between themselves in a more robust manner. Constrained environments such as those found in semiconductor manufacturing tools and enclosed glovebox environments are often cluttered with process tooling, posing significant obstacles for the manipulators. A strategy to deal with potential mutual collisions as well as potential collisions with tooling is needed and criteria must be quantified for all aspects of collision potential.

Beyond kinematics, hybrid twist-wrench screw theory offers a consistent method to model and control robotic devices [2]. Several key advantages exist. First a consistent means to control motion (twist) and forces and torques (wrench) which can be incorporated into the model for cooperating manipulators. Various cooperation strategies have been developed over the past two decades [3-11]. The need for dual-arm robots has been recognized in physical systems implementations [12,13]. In [14] dual-arm cooperative manipulation was examined for fixture interaction. Several experiments were also performed under a variety of application scenarios [15,16].

Automation in structured, static environments is technologically feasible. Similar tasks have been successfully automated in industry such as semiconductor manufacturing. One approach is to create single purpose automated tooling to augment the human and minimize exposure. This approach of course is not flexible once the process changes and therefore tooling requirements change. Robotics becomes attractive, as robots are easily adaptable to process changes when compared to fixed automation. Furthermore, dual-arm robots expand capability. There typically exists dedicated process tooling that is a required essential part of the process. At a minimum the robot must also be able to interact with the process tools. Special handling tooling is often also necessary. Thus, the use of additional flexible robotics alternatives in lieu of fixed automation can minimize tooling needed; i.e. a second manipulator to create a dual-arm robot.

Modular robots have been investigated for hazardous material applications using dual-arm robots [17]. A variety systems making use of commercially available robotic hardware has been investigated for many applications in hazardous material handling [18]. The use of a Staubli RX90 robot was investigated for a welding application in a cluttered recycling environment [19]. Dual-arm manipulators in telerobotic operations and applications are increasingly important [20] and the need to investigate their operations in cluttered environments is critical to successful application of these systems [21].

The effective use of dual-arm robotics for hazardous material handling, including the application of manipulating tooling with work-in-process has been investigated [15,16]. The benefit of using dual-arm robots is that much like humans, one arm can manipulate the work in process while the other manipulates a tool. There are additional obvious benefits such as sharing a heavy load. However, one of the most desirable features for these constrained environments is that dual manipulators are directly tractable to dual manual controllers for teleoperation.

The use of dual-arm robots has occurred in the semiconductor industry [22]. Thus far, due to the structured environment these are typically four degree of freedom robots (planar robots with an additional vertical axis). These are typically deployed in chamber tools. In this application the desire is to remove the human from the process so that the human does not contaminate the process. The trend in the semiconductor industry appears to be heading for more dexterous robots within the semiconductor tool. This is due to more elaborate configurations with an in-line approach to processing of the chambers as opposed to the rotary configuration.

III. TESTBED REQUIREMENTS

The flexibility offered by the use of dual-arm robotics becomes attractive if the challenging complexity of the environment can be managed. Ten requirements for a dual-arm robot are discussed below. The presentation of these requirements is to indicate areas for research in the testbed.

1. Compactness
2. Reliability
3. Dexterity
4. Collision Avoidance
5. Task Planning
6. Simulation
7. Accuracy and Repeatability
8. Teleoperation
9. Autonomous Operation
10. Force Control.

The robots themselves should be *compact*. This requirement makes modular automation attractive. However, if a manipulator of size scale, and dexterity is sufficiently compact unto itself, it becomes in essence a module. The robots to be used must be small, compact, and perhaps fold compactly when not in use to minimize interference in the environment. Yet they must also be able to span the workspace of the environment.

The ultimate decision for use of robotics and automation lies in the ability to enhance the *reliability* of the overall system. It is a given that the robot makes it safer for the human. Reliability is a serious concern in hazardous environments. The task plans can be developed to optimize the safety of the system by avoiding situations that place components near the edge of their performance envelopes. In the event of a component failure, replacing the affected component can repair the system and restore it to operation with less downtime and lower repair costs thus minimizing waste. Integration of an operational software and with fault-tolerance and condition based maintenance can further enhance the system reliability. The most complicated system components are embodied within the software architecture, necessitating a strict software reliability constraint. The individual software modules must be reliable in and of themselves, as well as when they are integrated on the particular platforms employed. Also, using two manipulators in a dual-arm robot increases overall hardware reliability since one may be used, if the other fails temporarily.

The *dexterity* refers to the configuration abilities of the manipulators in arbitrary position and orientation within the reachable workspace. It can be viewed as the maneuverability. The issues become increasingly complex when considering a pair of dexterous manipulators in a dual-arm robot. The combined and mutual dexterity must be considered for collision avoidance and path planning strategies. It is believed that human scale and dexterity can satisfy many situations occurring in glovebox and semiconductor tools.

The goal of *collision avoidance* is to permit a robot to work in an obstacle-strewn environment without damaging itself or any of the obstacles it encounters. This goal is especially relevant for systems that are used in constrained workspaces of gloveboxes and semiconductor tools. Obstacle avoidance in an obstacle-strewn environment is needed to assist the operator of a remote system and help avoid damaging expensive equipment or, even worse, causing further contamination of the environment from the hazardous materials. Furthermore, the robot must prohibit self-collision in the case of a dual-arm robot.

The automation of processing operations requires that complex tasks be deconstructed into a series of simple, “primitive” tasks in *task planning*. These primitive tasks represent the motions and operations assigned to individual components within the system. When coordinated, these primitive motions result in a complex, coordinated system motion. Ideally, task decomposition should be performed with the aid of an intelligent system, interfaced to the supervisor at a high level for teleoperated systems. The intelligent system will accept high-level instructions from the supervisor and use them to generate a task sequence and schedule, identify potential system conflicts, and allocate system resources to resolve those conflicts.

The assignment of primitive operations to individual components and the scheduling of these operations with respect to each other are important elements in developing efficient automation. The problem is similar in complexity to the problem of optimizing the operations of a manufacturing plant. Tasks are generally either independent or mutually dependent. Furthermore, tasks can be sequenced in serial, parallel or branching combinations. Using dual-arm robot further complicates the task-planning problem whether it is a shared task or not. The issues of inter- and intra-task sharing require further investigation in a dual-arm system. Thus, a hierarchy of rules governs the scheduling and sequencing of these tasks in the following categories:

- Path Planning
- Task Decomposition and Characterization
- Task Sequencing and Scheduling
- Task Sharing using Cooperative Manipulators
- Task Plan Verification
- Special Applications Modules (e.g. Tool Planning, Material Handling, Assembly/Disassembly).

Simulation offers the designer and user a window through which they can view the eventual system and establish confidence in a particular design or process operation. Simulation can also identify potential problems before they are discovered during implementation, and provide an environment where alternative designs can be freely explored. This work enhances the designer’s abilities to design new systems by enhancing their understanding of the system. Furthermore, simulations provide an environment for the training of operators and the examination of maintenance and contingency procedures. New equipment or procedures can be evaluated and validated prior to installation by these same personnel. Simulation not only enhances user training and designer understanding of the complexities of the system, but also can provide an invaluable environment for the simulation of components prior to their introduction into the system, and of potential reliability issues.

Nominal *accuracy* and *repeatability* of industrial robots are required for typical glovebox and semiconductor tool applications. Both are important. In autonomous operation sufficient repeatability is needed. Accuracy then depends on what the requirements of the task are and become important in teleoperation for precision tasks. The limitation of a remotely controlled dual-arm robot is the limiting factor of each of the manipulator’s accuracies.

It is a well-known fact that automation is very well suited for repetitive tasks in structured environments. Robots are typically preprogrammed to perform such tasks. This is plausible simply because the tasks and the environment are static and known beforehand. It becomes increasingly important to model the robot's surroundings in an unstructured environment. This helps to make automation an appropriate choice of operation. *Teleoperation* has been successfully applied to unstructured tasks such as space and underwater operations, nuclear facility maintenance and cleanup, and microsurgery. A telerobotic system consists of a human operator, a remote robotic system, and a man-machine interface. No matter how good the remote robot is, unless the man-machine interface is properly designed, the system will not perform well. A poorly designed man-machine interface can also introduce mental stress to the operator that will further deteriorate the system performance. Therefore, a man-machine interface plays a vital role in telerobotics, or teleoperation.

Autonomous operation is possible for well-structured tasks as mentioned above, this is true even in tightly confined spaces. Means to easily and seamlessly switch between teleoperation and autonomous operation are needed as the operator relinquishes control to the robot and vice-versa.

Force control is required for several reasons. First of all, central to the problem of cooperative manipulation, is the requirement that the dual-arm robot have the ability to know and control internal load distributions. This is critical in load sharing tasks to maintain integrity of the operational sequences as manipulators move within the workspace. Another important consideration is for bilateral manual controllers. In these systems forces and torques experienced at the end effectors are reflected back to the operator at the manual control station, thus the operator feels the work in process.

IV. TESTBED DESCRIPTION

A Cooperative Manipulation Testbed under development is described to investigate the complex scenarios associated with cooperative manipulation. The testbed development strategy is discussed including aspects of software and hardware.

The testbed is being developed includes two recently acquired Staubli RX60 CR clean room robots [23]. These are six degree-of freedom manipulators as shown in Figure 1. The reach of each of these manipulators is 665 mm. There also exists an optional longer reach version of 865 mm. The reach extends to front and back of the manipulators, creating an extended workspace. Furthermore, the manipulators can work cooperatively and perform similar functions to humans in both scale and dexterity. The robots have repeatability of ± 0.02 mm Also these robots could be mounted to translating tracks to increase the span of their workspaces.

Figure 1. Staubli RX60 CR Robot

Requisite tooling, end-effectors, and mock-up environments are being developed as the new system is being set-up in a new engineering lab space at the University of North Florida (UNF). General-purpose grippers and six-axis force sensors will be interfaced to the robots with manual controllers added later. The approach is to use the RX60 CR robots as the hardware platform (see Figure 2).

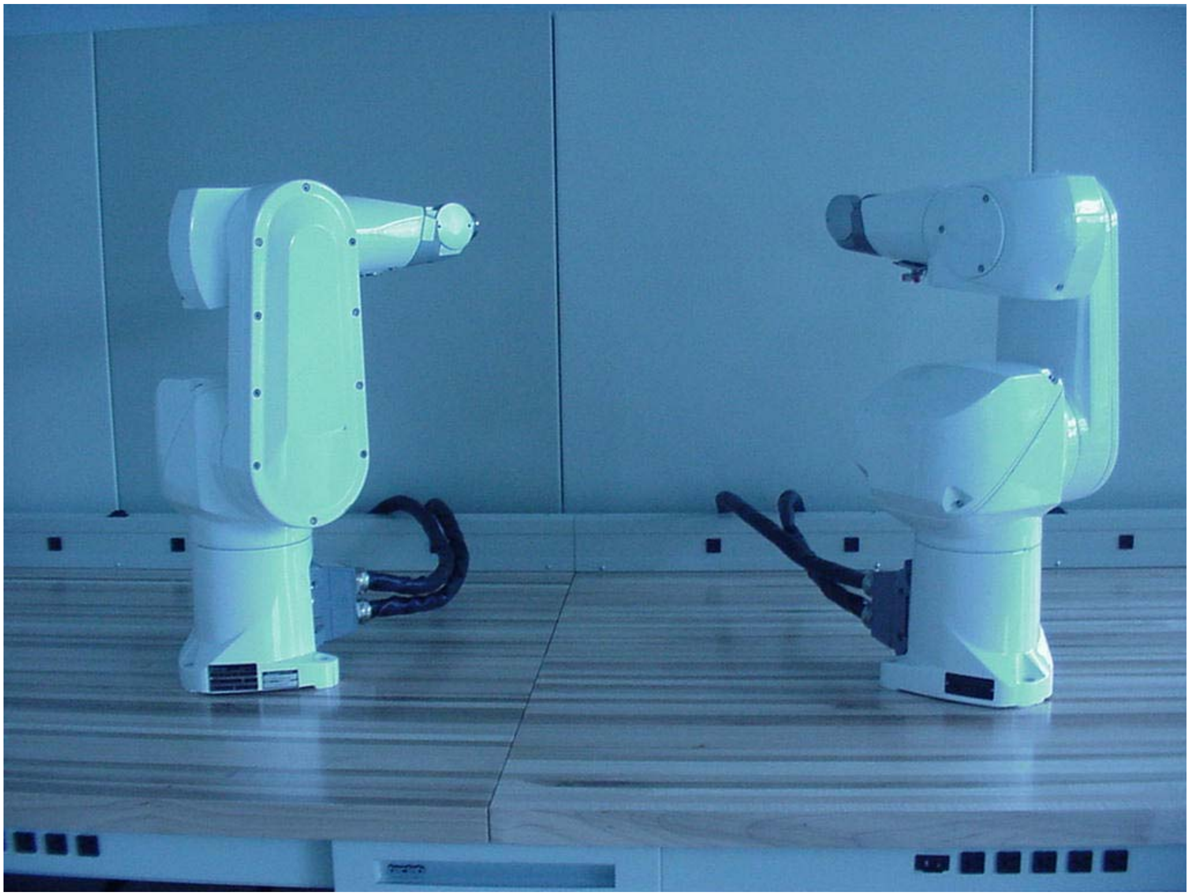

Figure 2. Staubli RX60 CR Dual-Arm Robot Testbed

Mock-up scenarios are being design to be rapidly set-up and configured using extruded aluminum components and fasteners The mock-ups are to be reconfigurable for a variety of demonstration scenarios in which a dual-arm robot is required. A corresponding programming and simulation environment is being developed with the testbed for off-line experimentation of algorithms, particularly the collision avoidance algorithms. The testbed software development strategy is discussed below.

The robot application language is V++, however this will be extended using OSCAR (Operational Software Components for Advanced Robotics) [24]. This object-oriented approach is an open architecture software system with standardized software interfaces enabling 3rd party software vendors, rapid software upgrades, and reconfiguration. The PC-based environment enables the embedded actuator joint-level servo control to create capabilities of kinematic control, force control, motion planning, obstacle avoidance, dual-arm and multi-arm cooperation. Throughout the development process, the currently existing base software classes from OSCAR will be extended with new software classes to support the new features for reconfigurable robot control including criteria-based decision making. The testbed is to contain the following core modules:

- Kinematics and Dynamics Software
- Redundancy Resolution Software
- Criteria Fusion Software
- Decision Making Software
- Collision Avoidance
- Cooperating Manipulation
- Scheduling and Coordination Interfaces
- System Reliability Extensions
- Real Time Communications.

These capabilities require the implementation of an operational software architecture and communications network that is capable of interfacing different equipment produced by different vendors including the Staubli robots and ancillary devices such as force sensors and application tools.

The costs associated with the implementation of a highly automated system have led to the increased demand for accurate simulations of these proposed systems prior system deployment. Such simulations can save time and money, while improving the reliability of the system. These simulations serve as valuable confirmations of the configuration management studies, the task planning, and the operational software and communications systems. The initial simulation base will use RoboWorks [25]. This is an economical package with features to allow animation and simulation of the dual-arm system in multiple complex application scenarios. A full suite of simulation configurations to match the reconfigurable hardware mock-ups is planned for development.

V. STAUBLI RX-60 KINEMATICS

The number of degrees of freedom that a manipulator possesses is the number of independent position variables, which need to be specified in order to locate all parts of the spatial mechanism. The Staubli RX60 CR robot is a six degree of freedom robot. Thus the robot under may in general arbitrarily orient (3 independent rotations) its end-effector at any position (3 independent translations) within its workspace.

Figure 3 shows the six rotary joints of the robot. Joint 1 rotates with axis perpendicular to base A. Joint 2 rotates perpendicular to Joint 1. Joint 3 rotates parallel to Joint 2 and is offset by the link indicated as C in Figure 3. Joint 4 the next joint in the kinematic chain is perpendicular to Joint 3, Joint 4 also intersects with Joint 5 and Joint 6 to form a "wrist" at the end of the manipulator. Therefore, Staubli RX60 CR robot has six degrees of freedom in a configuration similar to the well-known PUMA robot, however there are some differences.

Manipulators may be considered as a set of bodies, or links, connected in a kinematic chain by joints. Each joint of the robot exhibits one degree of freedom. Many manipulators have only revolute joints like the Staubli RX60 CR. Neighboring links have a common joint axis between them. Distance along the common axis from one link to the next is link offset, d_i. The amount of rotation about the common axis between one link and its neighbor is joint angle, θ_i. Any robot can be described kinematically by giving four quantities for each link; these describe the link and its connection with neighboring link. The definition of mechanisms by means of these quantities is a convention called Denavit-Hartenberg notation [26].

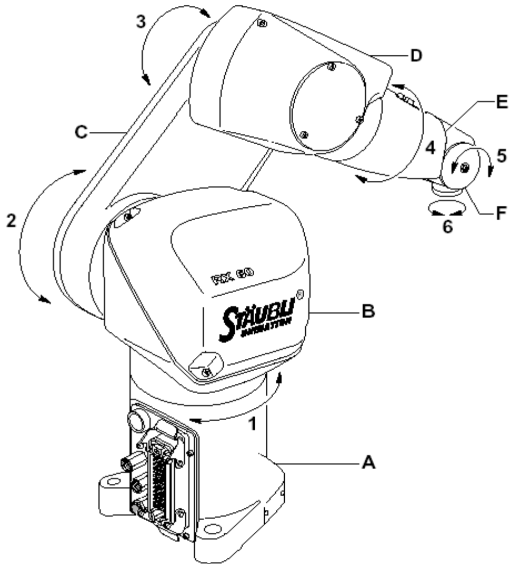

Figure 3. Staubli RX60 CR Robot Joints

For a six-jointed robot, 18 parameters are required to completely describe the fixed portion of its kinematics. The D-H parameters for the Staubli RX60 CR robot are provided in Table 1.

Table 1. D-H Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	0	θ_2
3	0	a_2	d_3	θ_3
4	-90	*0*	d_4	θ_4
5	90	0	0	θ_5
6	-90	0	0	θ_6

The constant D-H parameters for the robot are the twist angles α_{i-1}, the link lengths a_{i-1}, and the link offsets d_i, the variable joint angles θ_i. Table 2 provides the values for the non zero link lengths and link offsets.

Table 2. D-H Parameter Values

Parameter	Values (mm)
a_2	290
d_3	49
d_4	310

Given a set of joint angles, the forward kinematic problem is simply to compute the position and orientation of the tool frame relative to the base frame. Link transformations are computed as follows using homogeneous transformations.

$$
{}^{0}_{1}T = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 & 0 \\ s\theta_1 & c\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}_{2}T = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_2 & -c\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}_{3}T = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_2 \\ s\theta_3 & c\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}_{4}T = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ -s\theta_4 & -c\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{4}_{5}T = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_5 & c\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}_{6}T = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s\theta_6 & -c\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1)
$$

${}^{0}_{6}T$ is formed by matrix multiplication of the individual link matrices.

$$
{}^{4}_{6}T = {}^{4}_{5}T * {}^{5}_{6}T = \begin{bmatrix} c_5c_6 & -c_5s_6 & -s_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ s_5c_6 & -s_5s_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)
$$

Also

$$
{}^{3}_{6}T = {}^{3}_{4}T\,{}^{4}_{6}T = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5c_6 - s_4c_6 & c_4s_5 & 0 \\ s_5c_6 & -s_5s_6 & c_5 & d_4 \\ -s_4c_5c_6 - c_4s_6 & s_4c_5s_6 - c_4c_6 & s_4s_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)
$$

Because joints 2 and 3 are always parallel, multiplying ${}^{1}_{2}T$ and ${}^{2}_{3}T$ first and applying sum of angle formulas will yield a somewhat simpler final expression.

$$
{}^{1}_{3}T = {}^{1}_{2}T\,{}^{2}_{3}T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_2c_2 \\ 0 & 0 & 1 & d_3 \\ -s_{23} & -c_{23} & 0 & -a_2s_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4)
$$

Where sum of angles formulas are used, for example $c_{23} = c_2c_3 - s_2s_3$, $s_{23} = c_2s_3 + s_2c_3$. Then

$$
{}^{1}_{6}T = {}^{1}_{3}T\,{}^{3}_{6}T = \begin{bmatrix} {}^{1}r_{11} & {}^{1}r_{12} & {}^{1}r_{13} & {}^{1}p_x \\ {}^{1}r_{21} & {}^{1}r_{22} & {}^{1}r_{23} & {}^{1}p_y \\ {}^{1}r_{31} & {}^{1}r_{32} & {}^{1}r_{33} & {}^{1}p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)
$$

Where

$$
\begin{aligned}
{}^{1}r_{11} &= c_{23}\left[c_4c_5c_6 - s_4s_6\right] - s_{23}s_5s_6, \\
{}^{1}r_{21} &= -s_4c_5c_6 - c_4s_6, \\
{}^{1}r_{31} &= -s_{23}\left[c_4c_5c_6 - s_4s_6\right] - c_{23}s_5c_6, \\
{}^{1}r_{12} &= -c_{23}\left[c_4c_5c_6 + s_4c_6\right] + s_{23}s_5s_6, \\
{}^{1}r_{22} &= s_4c_5s_6 - c_4c_6, \\
{}^{1}r_{32} &= s_{23}\left[c_4c_5s_6 + s_4c_6\right] + c_{23}s_5s_6, \\
{}^{1}r_{13} &= -c_{23}c_4s_5 - s_{23}c_5, \\
{}^{1}r_{23} &= s_4s_5, \\
{}^{1}r_{33} &= s_{23}c_4s_5 - c_{23}c_5, \\
{}^{1}p_x &= a_2c_2 - d_4s_{23}, \\
{}^{1}p_y &= d_3, \\
{}^{1}p_z &= -a_2s_2 - d_4c_{23}. \qquad (6)
\end{aligned}
$$

Finally, to obtain the product of all six-link transforms

$$
{}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (7)
$$

Where

$$
\begin{aligned}
r_{11} &= c_1\left[c_{23}\left(c_4c_5c_6 - s_4s_6\right) - s_{23}s_5c_6\right] + s_1\left(s_5c_5c_6 + c_4s_6\right) \\
r_{21} &= s_1\left[c_{23}\left(c_4c_5c_6 - s_4s_6\right) - s_{23}s_5c_6\right] - c_1\left(s_5c_5c_6 + c_4s_6\right), \\
r_{31} &= -s_{23}\left(c_4c_5c_6 - s_4s_6\right) - c_{23}s_5c_6, \\
r_{12} &= c_1\left[c_{23}\left(-c_4c_5s_6 - s_4c_6\right) + s_{23}s_5s_6\right] + s_1\left(c_4c_6 + s_4c_5s_6\right), \\
r_{22} &= s_1\left[c_{23}\left(-c_4c_5s_6 - s_4c_6\right) + s_{23}s_5s_6\right] - c_1\left(c_4c_6 - s_4c_5s_6\right), \\
r_{32} &= -s_{23}\left(c_4c_5c_6 - s_4s_6\right) - c_{23}s_5c_6,
\end{aligned}
$$

$$r_{13} = -c_1\left(c_{23}c_4s_5 + s_{23}c_5\right) - s_1s_4s_5,$$
$$r_{23} = -s_1\left(c_{23}c_4s_5 + s_{23}c_5\right) + c_1s_4s_5,$$
$$r_{33} = s_{23}c_4s_5 - c_{23}c_5,$$
$$p_x = c_1\left[a_2c_2 - d_4s_{23}\right] - d_3s_1,$$
$$p_y = s_1\left[a_2c_2 - d_4s_{23}\right] + d_3c_1,$$
$$p_z = -a_2s_2 - d_4c_{23}. \qquad (8)$$

The above equations specify how to compute the position and orientation of Frame {6} at the wrist relative to Frame {0} at the base of the robot. Given the D-H parameters of Table 1 and Table 2, for any specified set of joint angles, the position and orientation of the Frame {6} located at the wrist is the forward kinematics.

The inverse kinematic problem is not as simple as the forward kinematics. In the case of the Staubli RX60 CR the closed-form analytic solution exists and one method is presented here similar to [27,28]. This is used as a starting point to begin to implement the kinematics for the Staubli robots. It is likely that to fully implement cooperative manipulation, the kinematics solution similar to [29] will be preferred. In both cases the purely algebraic method is used to solve Staubli RX-60 CR as opposed to numerical methods.

To get inverse kinematics solve,

$${}^0_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$= {}^0_1T(\theta_1)\,{}^1_2T(\theta_2)\,{}^2_3T(\theta_3)\,{}^3_4T(\theta_4)\,{}^4_5T(\theta_5)\,{}^5_6T(\theta_6) \qquad (9)$$

To solve first for θ_i when ${}^0_6\mathrm{T}$ is given as numeric values, the above equation can be restated as

$$\left[{}^0_1T(\theta_1)\right]^{-1}\,{}^0_6T = {}^1_2T(\theta_2)\,{}^2_3T(\theta_3)\,{}^3_4T(\theta_4)\,{}^4_5T(\theta_5)\,{}^5_6T(\theta_6)$$

Inverting ${}^0_1T$ is written as

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^1_6T \qquad (10)$$

Where ${}^1_6T$ is given by Equation 5. Equating the (2,4) elements, i.e. 2 row and 4th column of both matrices of Equation10 gives

$$-s_1p_x + c_1p_y = d_3 \qquad (11)$$

Trigonometric substitutions are made to solve above equation

$$p_x = \rho\cos\phi,$$
$$p_y = \rho\sin\phi, \qquad (12)$$

Where

$$\rho = \sqrt{p_x^2 + p_y^2},$$
$$\phi = A\tan 2\left(p_y, p_x\right). \qquad (13)$$

Substituting yields

$$c_1s_\phi - s_1c_\phi = \frac{d_3}{\rho}. \qquad (13)$$

where $\sin(\phi-\theta) = \cos\phi\sin\theta - \sin\phi\cos\theta$

and by applying the formula to Equation 13 gives

$$\sin(\phi-\theta_1) = \frac{d_3}{\rho}. \qquad (14)$$

Hence

$$\cos(\phi-\theta_1) = \pm\sqrt{1-\frac{d_3^2}{\rho^2}}, \qquad (15)$$

Therefore

$$\phi-\theta_1 = A\tan 2\left(\frac{d_3}{\rho}, \pm\sqrt{1-\frac{d_3^2}{\rho^2}}\right). \qquad (16)$$

Where θ_1 from above equation is now known,

$$\theta_1 = A\tan 2\left(p_y, p_x\right) - A\tan 2\left(d_3, \pm\sqrt{p_x^2 + p_y^2 - d_3^2}\right) \qquad (17)$$

There are two possible solutions for θ_1 corresponding to plus-or-minus sign in above equation. Equating (1,4) and (3,4) elements of Equation 5 gives

$$c_1p_x + s_1p_y = -d_4s_{23} + a_2c_2,$$
$$-p_z = d_4c_{23} + a_2s_2. \qquad (18)$$

Squaring Equations 11 and 18 and adding the resulting equations gives

$$-d_4s_3 = K, \qquad (19)$$

Where

$$K = \frac{p_x^2 + p_y^2 + p_z^2 - a_2^2 - d_3^2 - d_4^2}{2a_2}. \qquad (20)$$

Note that a_3 for the Staubli RX60 CR which differs from the PUMA solution [xx]. Equation 19 is of the form Equation 11 and so the same kind of trigonometric substitution to yield a solution for θ_3.

$$\theta_3 = A\tan 2\left(0, d_4\right) - A\tan 2\left(K, \pm\sqrt{d_4^2 - k^2}\right).$$

$$(21)$$

This leads to two different solutions for θ_3.

Next consider Equation 9, now rewriting it so that the left-hand side is a function of only known values and θ_2

$$\left[{}^{0}_{3}T(\theta_2)\right]^{-1}\,{}^{0}_{6}T = {}^{3}_{4}T(\theta_4)\,{}^{4}_{5}T(\theta_5)\,{}^{5}_{6}T(\theta_6), \quad (22)$$

or

$$\begin{bmatrix} c_1c_{23} & s_1c_{23} & -s_{23} & -a_2c_3 \\ -c_1s_{23} & -s_1s_{23} & -c_{23} & a_2s_3 \\ -s_1 & c_1 & 0 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{3}_{6}T \quad (23)$$

Where ${}^{3}_{6}T$ is given by Equation 3. Equating (1,4) and (2,4) elements on both sides of Equation 23 gives,

$$\begin{gathered} c_1c_{23}p_x + s_1c_{23}p_y - s_{23}p_z - a_2c_3 = 0, \\ -c_1c_{23}p_x - s_1s_{23}p_y - c_{23}p_z + a_2s_3 = d_4 \end{gathered} \quad (24)$$

These equations can be solved for s_{23} and c_{23}, resulting in

$$s_{23} = \frac{(-a_2c_3)\,p_z + (c_1p_x + s_1p_y)(a_2s_3 - d_4)}{p_z^2 + (c_1p_x + s_1p_y)^2},$$

$$c_{23} = \frac{(a_2s_3 - d_4)\,p_z - (-a_2c_3)(c_1p_x + s_1p_y)}{p_z^2 + (c_1p_x + s_1p_y)^2}. (25)$$

Sum of θ_2 and θ_3 is then

$$\theta_{23} = A\tan 2\begin{bmatrix} (-a_2c_3)\,p_z + (c_1p_x + s_1p_y)(-d_4 + a_2s_3), \\ (a_2s_3 - d_4)\,p_z - (-a_2c_3)(c_1p_x + s_1p_y) \end{bmatrix} \quad (26)$$

Equation 26 computes four values according to four possible combinations of solutions for θ_2 and θ_3. Then the four possible solutions for θ_2 are computed as

$$\theta_2 = \theta_{23} - \theta_3. \quad (27)$$

Equating (1, 3) and (3, 3) of equation 23, gives

$$\begin{gathered} r_{13}c_1c_{23} + r_{23}s_1c_{23} - r_{33}s_{23} = -c_4s_5, \\ -r_{13}s_1 + r_{23}c_1 = s_4s_5. \end{gathered} \quad (28)$$

As long as $s_5 \neq 0$, θ_4 is determined as

$$\theta_4 = A\tan 2(-r_{13}s_1 + r_{23}c_1, -r_{13}c_1c_{23} - r_{23}s_1c_{23} + r_{33}s_{23}). \quad (29)$$

When $\theta_5 = 0$, the manipulator is in a singular configuration in which joint axes 4 and 6 are collinear. This situation is detected by checking whether both arguments of the *Atan2* in Equation 29 are near zero, in which case the special configuration requires an exception handler.

Consider Equation 9 such that the left-hand side is a function of only known values and θ_4

$$\left[{}^{0}_{4}T(\theta_4)\right]^{-1}\,{}^{0}_{6}T = {}^{4}_{5}T(\theta_5)\,{}^{5}_{6}T(\theta_6), \quad (30)$$

Where $\left[{}^{0}_{4}T(\theta_4)\right]^{-1}$ is given by

$$\begin{bmatrix} c_1c_{23}c_4 + s_1s_4 & s_1c_{23}c_4 - c_1s_4 & -s_{23}c_4 & -a_2c_3c_4 + d_3s_4 \\ -c_1c_{23}s_4 + s_1c_4 & -s_1c_{23}s_4 - c_1c_4 & s_{23}s_4 & a_2c_3s_4 + d_3c_4 \\ -c_1s_{23} & -s_1s_{23} & -c_{23} & a_2s_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and ${}^{4}_{6}T$ is given by Equation 2. Equating (1,3) and (3,3) elements from both sides of Equation 30 gives

$$\begin{gathered} r_{13}(c_1c_{23}c_4 + s_1s_4) + r_{23}(s_1c_{23}c_4 - c_1s_4) - r_{33}(s_{23}c_4) = -s_5, \\ r_{13}(-c_1s_{23}) + r_{23}(-s_1s_{23}) + r_{33}(-c_{23}) = c_5 \end{gathered} \quad (31)$$

Solving for θ_5 as

$$\theta_5 = A\tan 2(s_5, c_5), \quad (32)$$

Where s_5 and c_5 are known from Equation 31. Applying same method to Equation 9 and equating (3,1) and (1,1) elements on both sides gives

$$\theta_6 = A\tan 2(s_6, c_6), \quad (33)$$

Where

$$\begin{gathered} s_6 = -r_{11}(c_1c_{23}s_4 - s_1c_4) - r_{21}(s_1c_{23}s_4 + c_1c_4) + r_{31}(s_{23}s_4), \\ c_6 = r_{11}[(c_1c_{23}c_4 + s_1s_4)c_5 - c_1s_{23}s_5] + r_{21}[(s_1c_{23}c_4 - c_1s_4)c_5 - s_1s_{23}s_5] \\ -r_{31}(s_{23}c_4c_5 + c_{23}s_5). \end{gathered} \quad (34)$$

These equations compute four different solutions because of positive and negative solutions appearing in Equations 17 and 21. There are four more solutions by “flipping” the wrist of the manipulator. For each of the solutions computed above, we get the flipped solutions by

$$\begin{gathered} \theta_4' = \theta_4 + 180^0, \\ \theta_5' = -\theta_5, \\ \theta_6' = \theta_6 + 180^0. \end{gathered} \quad (35)$$

After all eight solutions have been computed, some are all of them may have to be discarded because of joint limit violations. Of the remaining valid solutions, usually the one closest to the present manipulator configuration is chosen. Note that the kinematic solutions solve for the case of the wrist-partitioned manipulator. In other words, the location for the wrist with respect to the base is determined from which the end-effector kinematics are determined.

As an example consider θ_i values as

$$\begin{aligned} \theta_1 &= 0^o \\ \theta_2 &= -45^o \\ \theta_3 &= 45^o \\ \theta_4 &= 0^o \\ \theta_5 &= 0^o \\ \theta_6 &= 0^o \end{aligned} \tag{36}$$

Applying Equation 7 to above values and values from Table 1 and Table 2, gives

$${}^0_6T = \begin{bmatrix} 1 & 0 & 0 & 205.061 \\ 0 & -1 & 0 & 49 \\ 0 & 0 & -1 & -104.939 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Applying the inverse kinematics approach presented here to the above forward solution gives the original θ_i values. These are

$$\theta_1 = 0.0^o, -153.12^o$$

$$\theta_2 = -45.0^o, 80.80^o, -260.80^o, -135.0^o$$

$$\theta_3 = 45.0^o, 135.0^o$$

$$\theta_4 = 0.0^o, -180.0^o, 0.0^o, 0.0^o$$

$$\theta_5 = 0.0^o, 125.80^o, 125.80^o, 3.93^o$$

$$\theta_6 = 0.0^o, 26.88^o, 180.0^o, 139.1^o$$

The flipped solutions for θ_4, θ_5, θ_5 are

$$\theta_4' = 180.0^o, 0.0^o, 180.0^o, 180.0^o$$

$$\theta_5' = 0.0^o, -125.80^o, -125.80^o, -3.93^o$$

$$\theta_6' = 180.0^o, 206.88^o, 0.0^o, 319.1^o$$

From above the above values, the correct solution is chosen.

VI. CONCLUSION AND FUTURE WORK

The preliminary stages for development of a cooperative manipulation testbed have been provided. Ten requirements driving the technology for the testbed have been discussed. These drivers will continue to be used for the research and development objectives of the Cooperative Manipulation Testbed.

Central to the Cooperative Manipulation Testbed are a pair of Staubli RX60 CR robots. The integration of the robots with a software programming and simulation environment has been discussed. The testbed is being installed in the robotics laboratory in the new Science and Engineering Building at the UNF. The robotics and manufacturing laboratories are currently being brought on-line. Thus research and development using the Cooperative Manipulation Testbed as discussed in this paper will continue to evolve into the future.

The forward and inverse kinematic solutions for the Staubli RX60 CR robot have been provided in a first-pass approach using one of the conventional methods for closed-form algebraic solution, with a numerical example also provided. Further development of kinematic algorithms and algorithm modification will incorporate multiple solution checking as well as multiple manipulators. Clearly, a more extensive approach is needed to allow for cooperative manipulation and this effort will continue.

The algorithms for kinematics will also be verified using simulation. The kinematic algorithms will be ported to the Staubli RX60 CR robots for control of the manipulators. The kinematic algorithms presented in this paper will also be extended to facilitate coordination between the two robots working together with emphasis on the kinematics for the twelve degree-of-freedom dual-arm robot. The kinematics for the manipulators will also be extended with a strategy to specify the kinematics for one-object and two-object operations performed using the cooperating manipulators of the dual-arm robot. Further including aspects of the system software and hardware as well as the requisite tooling, end-effectors, and mock-up environments for initial demonstrations that fully utilize the kinematics described in the paper will be the subject of future reporting on the advances of the Cooperative Manipulation Testbed.

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